Institute of Information and Communication and Technology
Bangladesh University of Engineering and Technology

ICT 5103: Database Design and Management

Lecture 6

Instructor: Samin Rahman Khan

Relational Algebra

Chapter 4, Part A

Relational Query Languages

- **Query languages:** Allow manipulation and retrieval of data from a database.
- Relational model supports simple, powerful QLs:
  - Strong formal foundation based on logic.
  - Allows for much optimization.
- Query Languages != programming languages!
  - QLs not expected to be "Turing complete".
  - QLs not intended to be used for complex calculations.
  - QLs support easy, efficient access to large data set

Formal Relational Query Languages

- Two mathematical Query Languages form the basis for “real” languages (e.g. SQL), and for implementation:
  - **Relational Algebra:** More operational, very useful for representing execution plans.
  - **Relational Calculus:** Lets users describe what they want, rather than how to compute it. (Non-operational, **declarative**.)

Preliminaries

- A query is applied to relation instances, and the result of a query is also a relation instance.
  - Schemas of input relations for a query are fixed (but query will run regardless of instance!)
  - The schema for the result of a given query is also fixed! Determined by definition of query language constructs.
- Positional vs. named-field notation:
  - Positional notation easier for formal definitions, named-field notation more readable.
  - Both used in SQL

Example Instances

- "Sailors" and "Reserves" relations for our examples.
- We'll use positional or named field notation, assume that names of fields in query results are `inherited' from names of fields in query input relations.

R1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

S1

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

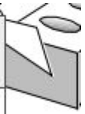
Sailors(<u>*sid:* `integer`</u>, *sname:* `string`, *rating:* `integer`, *age:* `real`)
Boats(<u>*bid:* `integer`</u>, *bname:* `string`, *color:* `string`)
Reserves(<u>*sid:* `integer`, *bid:* `integer`, *day:* `date`</u>)

Relational Algebra

- Basic operations:
  - **Selection (σ)** Selects a subset of rows from relation.
  - **Projection (π)** Deletes unwanted columns from relation.
  - **Cross-product (✕)** Allows us to combine two relations.
  - **Set-difference (—)** Tuples in reln. 1, but not in reln. 2.
  - **Union (∪)** Tuples in reln. 1 and in reln. 2.
- Additional operations:
  - Intersection, **join**, division, renaming: Not essential, but (very!) useful.
- Since each operation returns a relation, operations can be composed! (Algebra is "closed".)

Projection

- Deletes attributes that are not in projection list.
- Schema of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation.
- Projection operator has to eliminate duplicates! (Why??)
    - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it. (Why not?)



sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

$\pi_{sname,rating}(S2)$

age
35.0
55.5

$\pi_{age}(S2)$

Selection

- Selects rows that satisfy selection condition.
- No duplicates in result! (Why?)
- Schema of result identical to schema of (only) input relation.
- Result relation can be the input for another relational algebra operation! (Operator composition.)

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

$$\sigma_{rating>8}(S2)$$

sname	rating
yuppy	9
rusty	10

$$\pi_{sname,rating}(\sigma_{rating>8}(S2))$$

Union, Intersection, Set-Difference

- All of these operations take two input relations, which must be **union-compatible**:
  - Same number of fields.
  - `Corresponding' fields have the same type.
- What is the schema of the result?

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

$S1 \cup S2$

sid	sname	rating	age
22	dustin	7	45.0

$S1 - S2$

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$S1 \cap S2$

Cross-Product

- Each row of S1 is paired with each row of R1.
- Result schema has one field per field of S1 and R1, with field names \`inherited' if possible.
  - Conflict: Both S1 and R1 have a field called sid.

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

  - **Renaming operator:** $\rho(C(1\rightarrow sid1, 5\rightarrow sid2), S1 \times R1)$

Joins

- **Condition Join:** $R \bowtie_C S = \sigma_C(R \times S)$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	58	103	11/12/96

$$S1 \bowtie_{S1.sid < R1.sid} R1$$

- Result schema same as that of cross-product.
- Fewer tuples than cross-product, might be able to compute more efficiently
- Sometimes called a theta-join.

Joins

- **Equi-Join:** A special case of condition join where the condition c contains only **equalities**.

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/96
58	rusty	10	35.0	103	11/12/96

$$S1 \bowtie_{sid} R1$$

- Result schema similar to cross-product, but only one copy of fields for which equality is specified.
- **Natural Join:** Equijoin on all common fields.

Division

- Not supported as a primitive operator, but useful for expressing queries like:

    Find sailors who have reserved all boats.

- Let A have 2 fields, x and y; B have only field y:
    - A/B = $\{\langle x \rangle \mid \exists \langle x,y \rangle \in A \ \forall \ \langle y \rangle \in B\}$
    - i.e., **A/B contains all x tuples (sailors) such that for every y tuple (boat) in B, there is an xy tuple in A.**
    - Or: If the set of y values (boats) associated with an x value (sailor) in A contains all y values in B, the x value is in A/B.
- In general, x and y can be any lists of fields; y is the list of fields in B, and x y is the list of fields of A.

Examples of Division A/B

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

A

pno
p2

B1

pno
p2
p4

B2

pno
p1
p2
p4

B3

sno
s1
s2
s3
s4

A/B1

sno
s1
s4

A/B2

sno
s1

A/B3

Expressing A/B Using Basic Operator

- Division is not essential op; just a useful shorthand.
  - (Also true of joins, but joins are so common that systems implement joins specially.)
- Idea: For A/B, compute all x values that are not `disqualified' by some y value in B.
  - x value is disqualified if by attaching y value from B, we obtain an xy tuple that is not in A.

Disqualified x values: $\pi_X((\pi_X(A) \times B) - A)$

A/B: $\pi_X(A)$ – all disqualified tuple

Find names of sailors who've reserved #103

- Solution 1: $\pi_{sname}((\sigma_{bid=103}\ \text{Reserves}) \bowtie \text{Sailors})$

- Solution 2: $\rho\ (\text{Temp1}, \sigma_{bid=103}\ \text{Reserves})$

  $\rho\ (\text{Temp2}, \text{Temp1} \bowtie \text{Sailors})$

  $\pi_{sname}(\text{Temp2})$

- Solution 3: $\pi_{sname}(\sigma_{bid=103}\ (\text{Reserves} \bowtie \text{Sailors}))$

Find names of sailors who've reserved a red boat

- Information about boat color only available in Boats; so need an extra join:

$$\pi_{sname}((\sigma_{color='red'}\ Boats) \bowtie Reserves \bowtie Sailors\ )$$

- A more efficient solution:

$$\pi_{sname}(\pi_{sid}((\pi_{bid}\sigma_{color='red'}Boats) \bowtie Reserves) \bowtie Sailors)$$

A query optimizer can find this, given the first solution!

Find sailor who've reserved a red or a green boat

- Can identify all red or green boats, then find sailors who've reserved one of these boats:

$$\rho(\text{Tempboats}, (\sigma_{\text{color='red'} \vee \text{color='green'}}\text{Boats}))$$

$$\pi_{\text{sname}}(\text{Tempboats} \bowtie \text{Reserves} \bowtie \text{Sailors})$$

- Can also define Tempboats using union! (How?)
- What happens if $\vee$ is replaced by $\wedge$ in this query?

Find sailors who've reserved a red and a green boat

- Previous approach won't work! Must identify sailors who've reserved red boats, sailors who've reserved green boats, then find the intersection (note that sid is a key for Sailors):

$$\rho(\text{Tempred}, \pi_{sid}((\sigma_{color='red'}\text{Boats}) \bowtie \text{Reserves}))$$

$$\rho(\text{Tempgreen}, \pi_{sid}((\sigma_{color='green''}\text{Boats}) \bowtie \text{Reserves}))$$

$$\pi_{sname}((\text{Tempred} \cap \text{Tempgreen}) \bowtie \text{Sailors})$$

Find the names of sailors who've reserved all boats

- Uses division; schemas of the input relations to / must be carefully chose:

$$\rho\ (Tempsids, (\pi_{sid,bid} Reserves) \ / \ (\pi_{bid}\ Boats))$$

$$\pi_{sname} (Tempsids \bowtie Sailors)$$

- To find sailors who've reserved all 'Interlake' boats:

$$.....\quad / \ \pi_{bid}(\sigma_{bname='Interlake'}\ Boats)$$

Summary

- The relational model has rigorously defined query languages that are simple and powerful.
- Relational algebra is more operational; useful as internal representation for query evaluation plans.
- Several ways of expressing a given query; a query optimizer should choose the most efficient version.